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A new method for designing the heat exchangers constructed based on infinite regular polyhedrons geometry

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ABSTRACT

The paper presents the geometry of infinite regular polyhedrons that can be used to construct the heat exchange units. The basic structural component of the infinite solid consists of six hexagons, which meet in the common vertex. The mathematical model of heat and mass transfer inside the labyrinth heat exchanger is shown. The first and the second Kirchhoff's laws are used to simulate fluid flow distribution at hydraulic network created with connected basic solids. Heat balance equations are adopted to describe forced convection heat transfer inside developed heat exchanger. Good accordance among the experimental data and calculation results over whole simulation range is obtained. The efficiency and thermal characteristic of the labyrinth heat exchanger is presented in the form of graphs.

1. Introduction

The geometry of infinite regular polyhedrons is adopted by the author to construct the heat exchanger. The core of this device is formed by moving base element parallel to axis of Cartesian coordinate system. The basic structural component consists of six hexagons meet in the common vertex. This type of solid fills the space in the form of two congruent labyrinths. The present study focused on a mathematical model of heat and mass transport process in the heat exchanger, which geometry is shown in Fig. 1. The complex report about conception of utilization of infinite regular polyhedrons in engineering applications is presented in Ref. [1].

The thermal and pressure drop performance of heat exchange devices can be obtained by both numerical and analytical methods. In the last years, numerical simulations are more often used to investigate physical processes, which proceed in this kind of devices.

The control volume method is the most popular discretized algorithm for solving the governing differential equations that describe heat and mass transport phenomena. Some applications of this approximation technique are described below.

The numerical study of 3D laminar flow and heat transfer over a multi-row plate fin and tube heat exchanger are performed by Jang et al. [2]. Qu and Mudawar [3] experimentally and numerically investigated the fluid flow and heat exchange in a single-phase micro-channel heat sink fitted with a polycarbonate plastic cover plate. Radial heat flow pattern for rectangular and hexagonal fins of a heat exchanger is solved numerically by Perrotin and Clodic [4]. Numerical predictions of a flow characteristics of a horizontal air-cooled rectangular duct with square-sectioned cross-ribs mounted on its bottom surface are presented in [5]. The next following example of control volume formulation that is adopted for a heat exchanger design process is presented in [6]. A 3D mathematical model and its numerical approximation are applied to simulate the heat transfer augmentation in a circular tube with a helical turbulator in the form of a metallic wire by Makiharju et al. CFD simulation is employed to investigate the turbulent flow structure inside the header of a plate fin heat exchanger by Wen et al. and results of calculations are presented in Ref. [7]. Kasagi et al. used a commercial CFD code to calculate the thermal and velocity field around in-line tube bundles with three rows in the transverse direction and 10 columns in the longitude direction. Results of calculations are employed to train the neural network and to construct the heat transfer and pressure drop debase [8].

The finite element method is less popular for predicting the thermal and flow performance of heat exchangers. This type of a numerical analysis technique is adapted by Bejan et al. [9,10] to study of the optimal spacing between cylinders in a fixed volume cooled by natural and forced convection.

A mathematical model for predicting the steady-state thermal performance of one-dimensional multistream heat exchangers was solved analytically by Luo et al. [11]. They introduced three matching matrices: interchannel, entrance and exit. The solution can be also applied to the synthesis of heat exchanger networks. By introducing the correction factor of logarithmic mean

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Nomenclature

$\dot{C}$	flow-stream capacity rate, W/K
c_p	fluid specific heat at constant pressure, J/(kg K)
F	heat transfer surface area, m^2
$\dot{G}$	mass flow rate, kg/s
$\dot{G}o$	charge (load) mass flow rate, kg/s
l	hexagon side length, m
$\dot{Q}$	heat rate, W
R	local flow resistance, Pa s^2/kg^2
U	overall heat transfer coefficient, W/(m^2 K)
θ	fluid temperature, K

Greek symbols

Δ	pressure or temperature difference, Pa or K
δ	flow rate calculation error, kg/s
ε	temperature efficiency, %

Subscripts

b	base solid
C	cold fluid
H	hot fluid
i	number of node (cell) or iteration
in	inflow
j	cold fluid, number of branch
k	number of adjacent nodes,
m	number of branch at loop, measurements
n	number of equations
out	outflow
s	simulation

temperature difference the developed method can be valid for any types of two-stream heat exchangers. In Ref. [12] Ghosh et al. analyzed multistream heat exchangers. They applied the successive partitioning and area splitting methods for creating practical algorithm for design of multistream plate fin heat exchangers. The proposed method has also been verified by using the experimental results and a good agreement was obtained.

Problem of mass flow and conduction in inhomogeneous systems has been investigated by Dul'nev and Novikov [13–15]. Their model, which integrates both method of reduction to elementary components and theory of fluid flows, can be applied to analyze the performance of compact heat exchangers with complicated channel shapes.

Experimental results are used to verify the accuracy of the numerical solutions that are described above. For the most investigations, results of simulations agree with physical reality. However, algorithms of numerical analysis are CPU time demanding. Besides that, any interference to flow domain will require building a new computational model. From these reasons, both control volume method and finite element formulation are not useful to design the construction of heat exchangers.

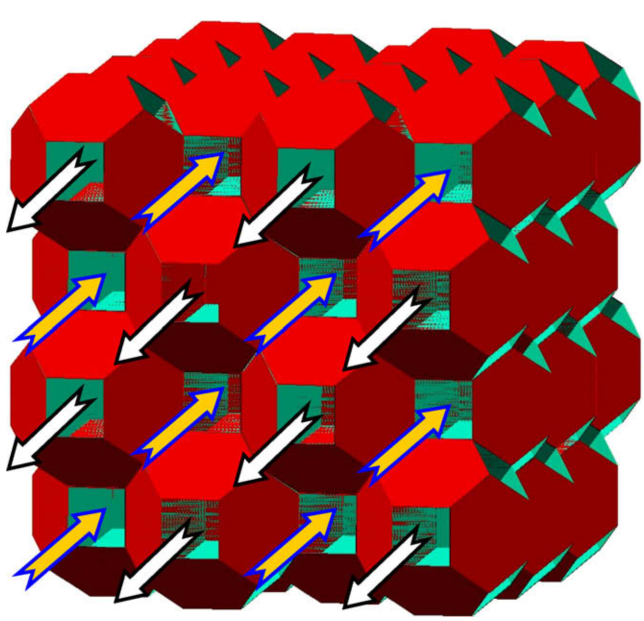

Fig. 1. The geometry of infinite regular polyhedrons.

The labyrinth heat exchanger is the conventional direct-transfer unit, because working fluids are separated by the heat transfer surface. There are some following analytical methods for designing this type of exchanger system: LMTD (log-mean temperature difference), ε-NTU (number of heat transfer units), $\mathbf{P}$-NTU$_t$ and ψ-$\mathbf{P}$. Detailed characteristics and routines for these design approaches are widely described in the literature (like in Refs. [16–21]) and for this reason, they are not discussed here.

The two idealizations that limit of these methods are not valid for the analyzed application. Firstly, heat transfer conditions are not uniform throughout the exchanger and secondly, the flow rate of fluids is not constant in each cell. Therefore, it was necessary to create a new method for designing the heat exchanger with the labyrinth core. A numerical algorithm for finding solution of this problem is the objective of the current paper.

2. Mathematical model of heat and mass transfer

It should be noted that each solid figure (see Fig. 2), which is a base component of the infinite regular polyhedron, represents single node of hydraulic network with zero connecting branches (pipes) length.

It is possible to distinguish two identical spatial loop configurations with virtual connections. The first network represents the flow geometry of a colder medium and the second represents the warmer medium flow geometry. We can detect the difference only in opposite flow directions. A fragment of the hydraulic network is presented in Fig. 3.

For mathematical analysis, it is allowed to separate the repeated module, shown in Fig. 4. This separated fragment is limited by two symmetry planes, which are parallel to the flow direction.

In order to characterize the flow distribution in this hydraulic system we can apply Kirchhoff's lows. The first Kirchhoff's low states that the sum of currents flowing towards the point is equal to the sum of currents flowing away from that point. By this definition, for incompressible fluid, mass conservation equation at each network receives the following form:

$$\sum_{i=1}^{k} \mathrm{sign}_j \dot{G}_j + \mathrm{sign}_i \dot{G}o_i = 0. \qquad (1)$$

It is assumed that when fluid flows in the direction to the node (cell) $\mathrm{sign}_i = \mathrm{sign}_j = +1$ and when it flows out $\mathrm{sign}_i = \mathrm{sign}_j = -1$. If there is no connection between nodes, sign_i equals to 0.

The second Kirchhoff's law states that the sum of the pressure drops around any loop of the channels must be equal to zero, which gives:

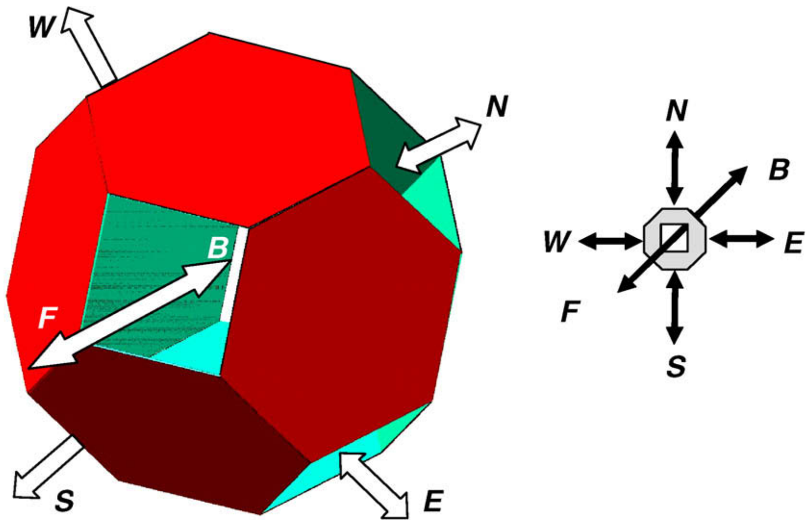

Fig. 2. The possible flow directions across single node (cell).

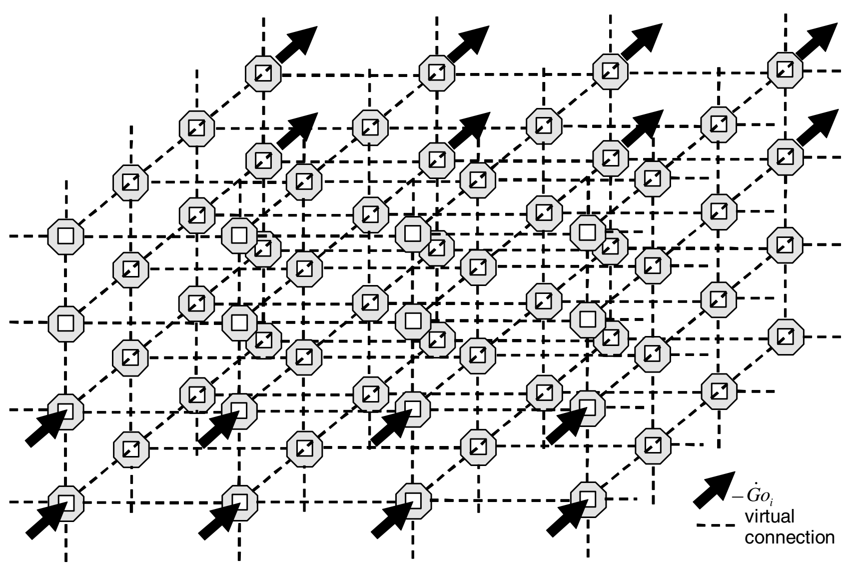

Fig. 3. The scheme of the analyzed flow arrangement for the one fluid.

$$\sum_{j=1}^{m} \mathrm{sign}_j \Delta p_j = 0. \tag{2}$$

For the analyzed hydraulic network the pressure drop of each branch is composed only of local losses and should be equal to:

$$\Delta p_j = R_j \dot{G}_j^2. \tag{3}$$

Forced convection heat transfer is observed in each cell. Hence, the author introduces, presented below, another balance equation valid for all nodes.

$$\sum_{i=1}^{k=6} \mathrm{sign}_i \dot{G}_i c_\mathrm{p} \theta_i - \sum_{j=1}^{l=8} U_j F_j \theta_\mathrm{H} + \sum_{j=1}^{l=8} U_j F_j \theta_j = 0. \tag{4}$$

The heat transfer area of the base solid F_j, as a function of a hexagon side length (see Fig. 5), can be calculated form the following relation:

$$F_\mathrm{b} = 8\frac{3\sqrt{3}}{2} l^2. \tag{5}$$

3. Numerical solution

As discussed in [22] several methods have been developed for computing the flow and pressure field of the working fluid. It is possible to mention three algorithms for network analysis: loop method, flow method and nodal method. As demonstrated in [23] first two methods give better convergence than the node pressure formulation. In the current work, the loop formulation is adopted for analysis of flow characteristic in multiple circuit configurations. The algorithm solves nonlinear Eq. (2) for corrective flow rates at each hydraulic loop. This method guarantees good convergence characteristics and low sensitivity with respect to starting data.

Flow and temperature distribution inside the heat exchanger in steady-state conditions are described by two sets of linear algebraic equations.

The first system of equations, in matrix form, is formulated by mass conservation law, according to Eq. (1):

$$\mathbf{A} G_j = \mathbf{B}. \tag{6}$$

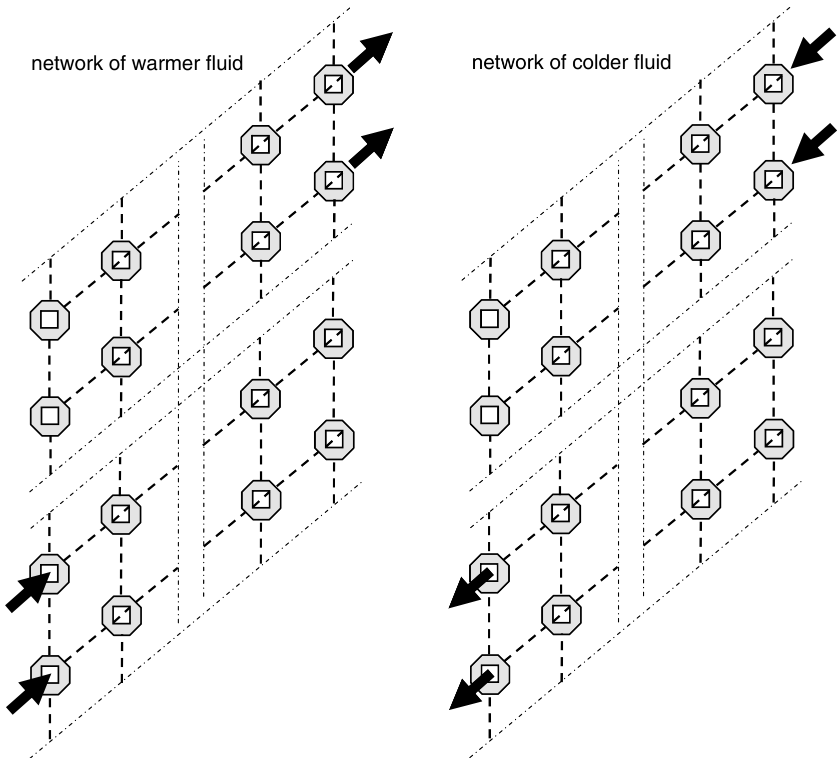

Fig. 4. The hydraulic network of the repeated module.

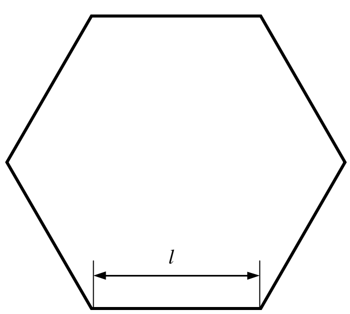

Fig. 5. Regular hexagon is a 1/8 of a single cell.

The coefficients of matrix **A** equal to zero or sign_j and coefficients of matrix **B** equal to zero or $\text{sign}_i \dot{G}o_i$.

The second system of equations is build by energy conversion law, according to Eq. (4).

$$\mathbf{C}\theta = 0. \tag{7}$$

The coefficients of matrix **C** equal to zero, $\text{sign}_i \dot{G}_i c_p$ or $U_i F_i$.

The nonlinear Eq. (2) can be expanded in Taylor series. If we take just only the first linear term, we obtain the third linear system of Eq. (8), which describes the pressure drop at each hydraulic loop.

$$\mathbf{D}G_j = \mathbf{E}. \tag{8}$$

The coefficients of matrix **D** can be equal to zero or $2R_j\dot{G}_j^0$ and coefficients of matrix **E** equal to zero or $R_j\left(\dot{G}_j^0\right)^2$.

Variable $\dot{G}_j^0$ is the result of calculation at iteration number $i-1$ and serves as the starting value of the flow rates at the next iteration number i.

The first system of equations (Eq. (5)) and the third (Eq. (8)) should be solved together by applying the Gaussian Elimination algorithm. The resulting flow distribution in heat exchanger core is only approximate. The iteration procedure is repeated until the mass balance will be less than the assumed error δ. In practice, four iterations give sufficient accuracy of results. After calculation of flow field inside heat exchanger we should to solve the second system of linear Eq. (7) to satisfy energy balance for each cell. As a result of calculation, we obtain working fluids temperature profiles along the heat exchanger. Brief overview of the solving procedure is presented in Fig. 6.

The discussed here algorithm was implemented as a computer code and it can be applied to optimize the flow and thermal performance of the heat exchanger, which construction is based on polyhedrons geometry.

In the author's opinion, this way of characterizing the flow and temperature distribution inside the heat exchanger has not been reported in the literature yet.

4. Validation test of the model

The experimental data are compared with results of calculations to verify the accuracy of the developed model. The measurements report is presented in Ref. [1]. The fragment of the developed heat transfer unit is shown in Fig. 7.

Calculations are performed for air temperature difference, which is changed between 5 °C and 28 °C. Reynolds number for air flow is applied in the range of 4300–8500. Fig. 8 shows value of relative error (Eq. (9)) as a result of a comparison between measured and predicted heat transfer rate of the tested device.

$$\text{ep} = \frac{\dot{Q}_m - \dot{Q}_s}{\dot{Q}_m} \times 100\%. \tag{9}$$

Relative maximum error (RME), given by Eq. (10), and root mean square error (RMSE), defined by Eq. (11), are adapted to quantify the divergence among measurement data and computer simulation results.

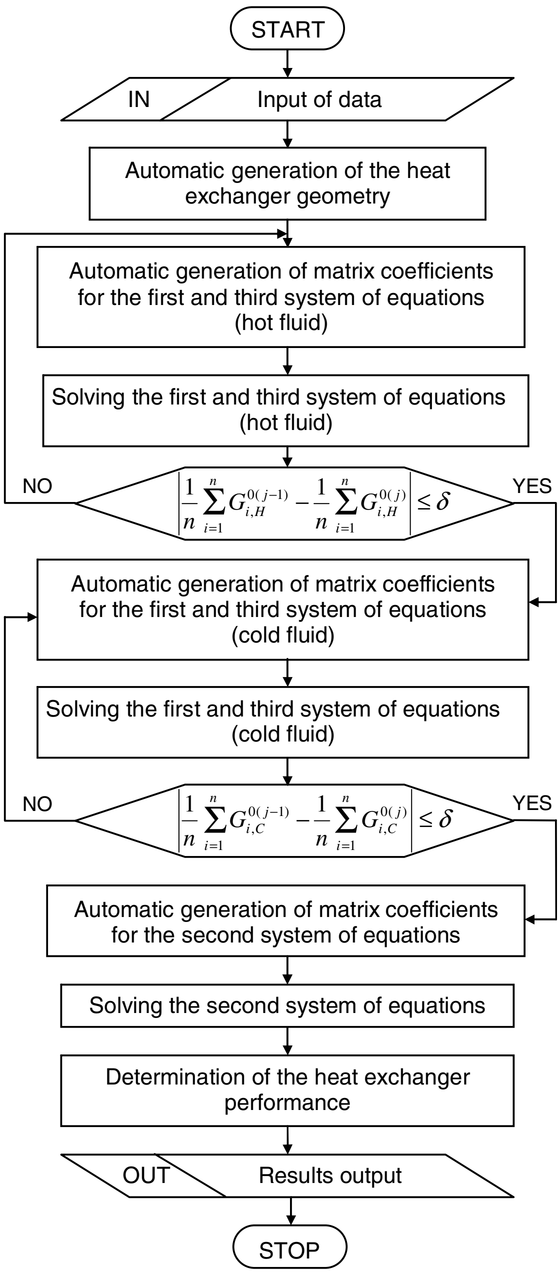

Fig. 6. The flow chart of the calculation scheme.

$$\mathrm{RME} = \max_{i=1,2,\ldots,n}\left\{\left|\frac{\dot{Q}_m - \dot{Q}_s}{\dot{Q}_m}\right| \times 100\%\right\}, \tag{10}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(\dot{Q}_m - \dot{Q}_s)^2}{k_e}}, \tag{11}$$

where $k_e = n - 1$ if $n < 30$ or $k_e = n$ if $n > 30$.

The relative maximum error of heat transfer rate is approximately 10.5%, while the average accuracy, described by RMSE, is equal to 6.5 W. For this reason, it can be concluded that the mathematical model and its numerical solution give an acceptable accuracy over whole simulation range. A very good approximation of the thermal characteristic is obtained for *Re* number ranged from 4300 to 6700. The results of validation test lead to conclusion that the computer code can be used successfully to predict the thermal performance of labyrinth heat exchangers.

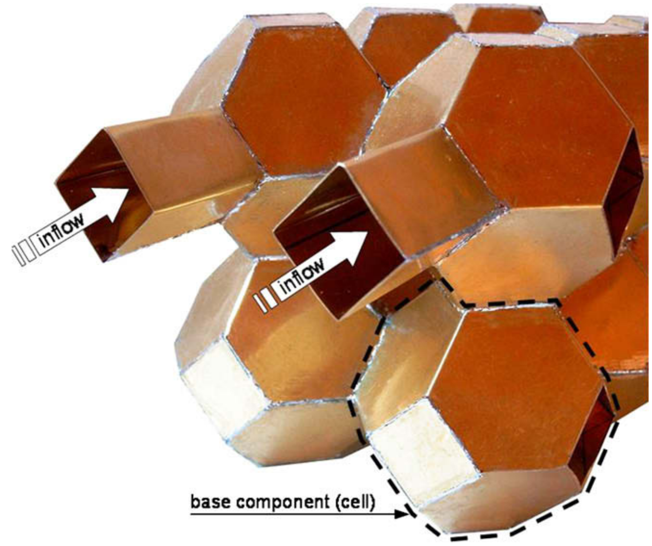

Fig. 7. The experimentally tested labyrinth heat exchanger.

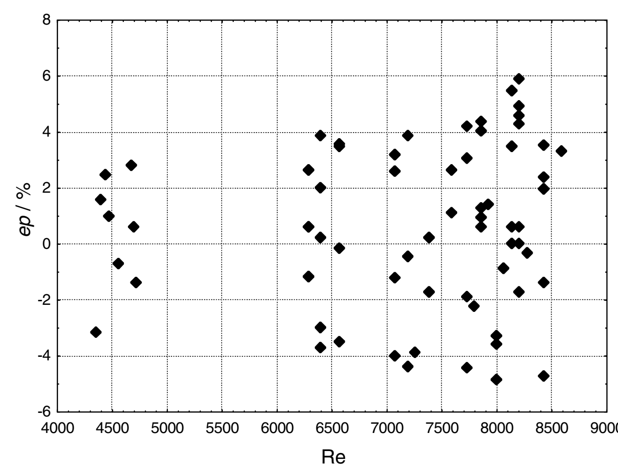

Fig. 8. Relative error as a function of Reynolds number.

5. Results of calculations

The thermal characteristic of the labyrinth heat exchanger is determined for the practical case – the same heat capacity of both medium flows:

$$\dot{C}_{\min} = \dot{C}_{\max} = \dot{C}_{\mathrm{H}} = \dot{C}_{\mathrm{C}}, \tag{12}$$

where $\dot{C} = \dot{G}\, c_p$.

Symbols, used in the current analysis, are shown in Fig. 9.

Fig. 10 shows the distribution of the flow rate of hot and cold fluids for the vertical cross-section of the heat exchanger.

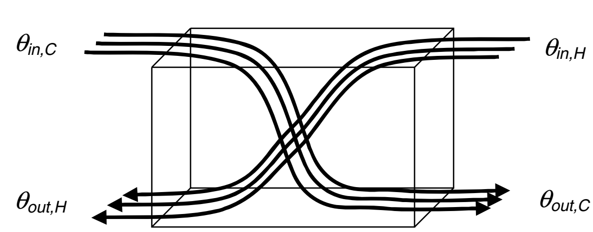

Fig. 9. Labyrinth heat exchanger flow paths.

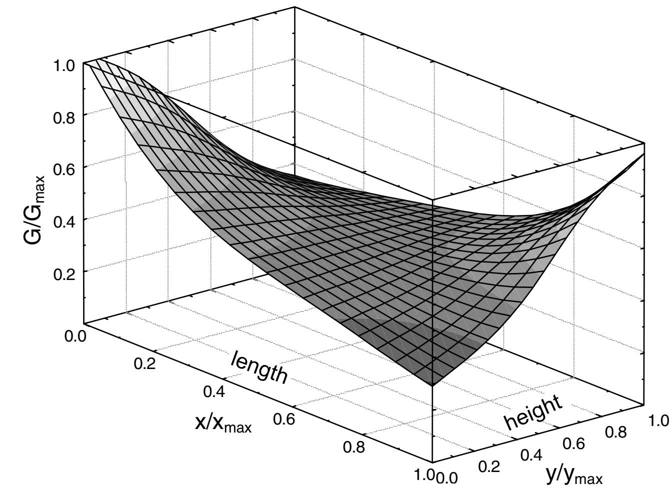

Fig. 10. Distribution of the flow rate for the vertical cross-section.

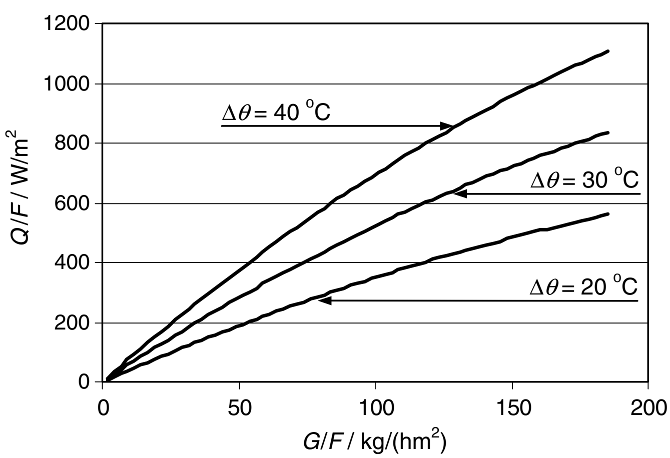

Fig. 11. The dependence of heat recovery on flow rate for three temperature differences between warm and cold fluid.

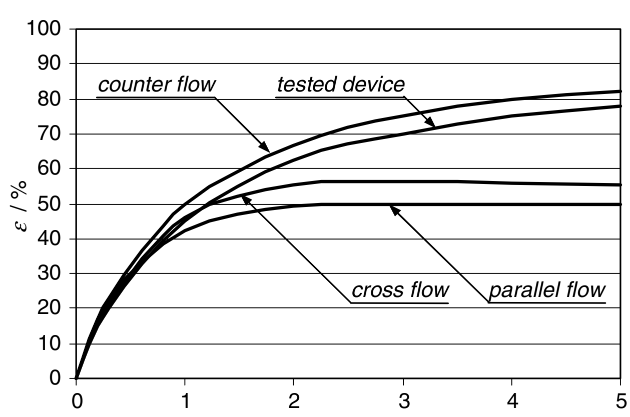

Fig. 12. Heat transfer efficiency as a function of heat transfer units for different flow configurations.

As seen in this figure, the heat transfer conditions are not the same along the device. And, as previously noted, it confirms that the use of traditional methods for designing this kind of heat exchanger, such as NTU and LMTD approaches, is not correct.

Heat transfer rate exchange between fluids depends on flow rate, temperature difference and area of a heat transfer surface. Based on the graphs in Fig. 11 we can size the labyrinth heat exchanger.

The performance of heat exchanger can be obtained by the following equation:

$$\varepsilon = \frac{\theta_{\mathrm{out,C}} - \theta_{\mathrm{in,C}}}{\theta_{\mathrm{in,H}} - \theta_{\mathrm{in,C}}} \times 100\%. \tag{13}$$

The dependence of the temperature efficiency ε (Eq. (13)) on number of heat transfer units is shown in Fig. 12.

As Fig. 12 shows, the efficiency of labyrinth heat exchanger is better than cross and parallel flow heat exchangers and is comparable to counter flow device.

6. Summary and concluding remarks

The new approach for designing labyrinth heat exchangers is necessary because heat transfer conditions are not uniform throughout the proposed device and the flow rate of fluids is not constant in each cell. The current paper describes the method, which solves this problem by adaptation of loop formulation for analysis of flow characteristic.

Accurate accordance between measurement data and computer simulation results is confirmed. So, the mathematical model and its numerical solution can be used successfully to predict the thermal performance of the developed type of heat exchangers.

Good temperature efficiency leads to the conclusion that core geometry, based on infinite regular polyhedrons, can be successfully applied, for example, in ventilation air heat recovery units.

Future development will focus in an intensification of the heat exchange inside presented construction and in new flow configurations.

Acknowledgements

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